On the strangeness content of the nucleon

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We revisit the classical relation between the strangeness content of the nucleon, the pion-nucleon sigma term and the $SU(3)_F$ breaking of the baryon masses in the context of covariant chiral perturbation theory. In particular, we consider the contributions of the decuplet resonances explicitly. We find that a value of the pion-nucleon sigma term of ∼60 MeV is not at odds with, but favored by the fulfillment of the Zweig rule. We compare these results with earlier ones and discuss the convergence of the chiral series as well as the uncertainties of chiral approaches to the determination of the sigma terms.

I. INTRODUCTION

The nucleon sigma terms, $\sigma_{\pi N}$ and σ_s, are observables of fundamental importance which embody the internal scalar structure of the proton and neutron. These are defined in the isospin limit ($m_u = m_d \equiv \hat{m}$) as[1]

$$
\begin{aligned}
\sigma_{\pi N} &= \frac{1}{2M_N}\langle N|\hat{m}\left(\bar{u}u + \bar{d}d\right)|N\rangle, \\
\sigma_s &= \frac{1}{2M_N}\langle N|m_s\bar{s}s|N\rangle,
\end{aligned}
\tag{1}
$$

and contain information on the contribution of the quark-condensate to the masses of the baryons. Besides being an essential piece to the understanding of the mass of the ordinary matter, $\sigma_{\pi N}$ and σ_s appear in the hadronic matrix elements of the (spin-independent) neutralino-nucleon elastic scattering cross section. An accurate determination of the sigma terms is then essential to constrain the parameter space of the underlying supersymmetric models from the experimental bounds in direct searches of weakly interacting dark matter particles [1].

It was first pointed out by Cheng that the contribution of the strange quark to the nucleon mass can be related with $\sigma_{\pi N}$ and the $SU(3)_F$-breaking of the baryon masses in the octet [2]. Namely, one can re-express the pion-nucleon sigma term as [3]

$$
\sigma_{\pi N} = \frac{\sigma_0}{1-y}, \tag{2}
$$

where y is the so-called "strangeness content" of the nucleon,

$$
y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle} = \frac{2\hat{m}\sigma_s}{m_s\sigma_{\pi N}} = 1 - \frac{\sigma_0}{\sigma_{\pi N}}, \tag{3}
$$

and

$$
\sigma_0 = \frac{\hat{m}}{2M_N}\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle. \tag{4}
$$

Thus, σ_0 can be understood as the value of the pion-nucleon sigma term in case that the strange-quark contribution to the nucleon wave function is null (Zweig rule [2]). These relations are very important in sigma-term physics as they imply strong constraints between $\sigma_{\pi N}$ and y. In particular, requiring the Zweig rule to be approximately true ($y \approx 0$) one obtains

$$
\sigma_{\pi N} \approx \sigma_0. \tag{5}
$$

---

[1] We use the normalization of the nucleon spinors $\bar{u}(\vec{p}', s')u(\vec{p}, s) = 2M_N\delta_{s's}$, with M_N the nucleon mass.

Now, assuming linear $SU(3)_F$ breaking, one can write σ_0 in terms of the physical masses of the baryon octet [2] leading, in combination with Eq. (2), to

$$\sigma_{\pi N} \simeq \frac{\hat{m}}{m_s - \hat{m}} \frac{(M_\Xi + M_\Sigma - 2M_N)}{1-y} \simeq \frac{27}{1-y}\ \mathrm{MeV}, \tag{6}$$

where we used $m_s/\hat{m} = 26(4)$ [4]. The Eq. (6) is an estimate obtained at leading order in a $SU(3)_F$-breaking expansion of the baryon-octet masses. At first glance and given the success of the Gell-Mann-Okubo (GMO) relation $3M_\Lambda + M_\Sigma - 2M_N - 2M_\Xi = 0$, which is obtained at the same order in the expansion and is verified experimentally at the few-percent level, one expects this result to be a very good approximation. However, it is important to confront Eq. (6) with calculations of higher order terms in the $SU(3)_F$-breaking expansion.

This can be done in a model-independent fashion using baryon chiral perturbation theory (BχPT), which is the effective field theory of QCD at low energies [5] and in the baryon sector [6] (for reviews see Refs. [7]). At leading order in the chiral expansion one recovers the linear $SU(3)_F$-breaking corrections to the baryon masses and the GMO relation, whereas at next-to-leading-order (NLO), one obtains a contribution coming entirely from a chiral loop, so it is a pure prediction of BχPT (see e.g. the recent analysis in Ref. [8]). The NLO chiral correction to σ_0 was first calculated by Gasser in Ref. [3] giving $\sigma_0 = 35(5)$ MeV, although this calculation was done in the context of a chiral model of the meson cloud around the baryon which considered contributions from virtual octet baryons only (and not from decuplet resonances). Later, Borasoy *et al.* performed a calculation of the baryon masses and σ_0 in heavy-baryon (HB) [9] expansion and up to next-to-next-to-leading order (NNLO) [11]. In this work, the contributions of the decuplet resonances were not implemented explicitly but through resonance-saturation hypothesis they contributed to several of the many low-energy-constants (LECs) appearing at this order. All in all, they reported the value $\sigma_0 = 36(7)$ MeV, which was almost identical to the NLO result obtained by Gasser 15 years earlier.

The agreement between these two different chiral calculations would suggest that the determination of σ_0 in BχPT is under good theoretical control. Furthermore, its value is slightly below the canonical result $\sigma_{\pi N} \simeq 45$ MeV [10], that was obtained by Gasser *et al.* from the πN elastic scattering data,[2] leading to a strangeness content in the nucleon in accordance with a rather small violation of the Zweig rule. This picture, however, has been challenged by recent studies based on modern πN scattering data. Indeed, partial-wave analyses of the enlarged πN database carried out by the George-Washington University group [14], resulted in larger values of the pion-nucleon sigma term, $\sigma_{\pi N} = 64(8)$ MeV [15]. Besides that, a study of πN elastic scattering in BχPT reveals that modern partial-wave analyses are, in general, more consistent with different scattering phenomenology than the older ones and lead to a relatively large value of the sigma-term, cf. $\sigma_{\pi N} = 59(7)$ MeV [16]. These values of $\sigma_{\pi N}$, together with the current one of $\sigma_0(\sim 35$ MeV$)$, imply a strangeness contribution to the mass of the nucleon of $\sim$300 MeV. Although not impossible, such a scenario with a strong breaking of the Zweig rule is theoretically implausible, moreover after the experimental evidence pointing to a negligible strangeness contribution in other properties of the nucleon such as its electromagnetic structure [17] and spin [18].

In order to solve this puzzle the stress has been put, so far, on the value of the pion-nucleon sigma term. Inconsistencies and problems within the experimental πN database has been scrutinized (see e.g. Ref. [19]) and the systematic errors in the relation between the isoscalar scattering amplitude and $\sigma_{\pi N}$ carefully investigated [16, 20]. Little attention has been given to the value of σ_0 though. There are two main lines along which the present determinations can be examined. On one hand, the result of Gasser [3] can be put in a more evolved context than the one available at that time, whereas the one of Borasoy *et al* [11] might be afflicted by the poor convergence of the chiral series shown by HB in the $SU(3)_F$ theory [8, 21]. On the other, none of the two calculations include the *explicit* contributions of the decuplet resonances. These have to be introduced in $SU(3)_F$-BχPT due to the fact that the typical scale for the onset of the decuplet resonances $(M_T - M_B)/\Lambda_{\chi SB} \sim 0.3$ is well below the chiral expansion parameter $p = m_K/\Lambda_{\chi SB}$.

A suitable approach to tackle both issues is a covariant formulation of BχPT with a consistent power-counting via the extended-on-mass-shell renormalization (EOMS) scheme [22]. The relativistic corrections included in this approach have been shown to tame the poorly convergent series of the HB expansion in baryonic observables as important as the magnetic moments [21, 23] or masses [8, 24]. Moreover, once a prescription is taken to treat the problem of the interacting Rarita-Schwinger fields [25], this scheme is straightforwardly applicable to include the contributions of the decuplet resonances [23]. In this work we calculate σ_0 up to NLO using covariant BχPT renormalized in the EOMS prescription and including explicitly the effects of the decuplet. We will compare the results with those obtained in the HB expansion and we estimate systematic higher-order effects through a partial calculation of NNLO pieces. All together, we will find that the effect of the decuplet is sizable and that it pushes the value of σ_0 upwards making the

[2] For a detailed exposition of the dispersive methods for obtaining $\sigma_{\pi N}$ from the analytic continuation of the πN scattering amplitude to the Cheng-Dashen point see Refs. [10, 12, 13].

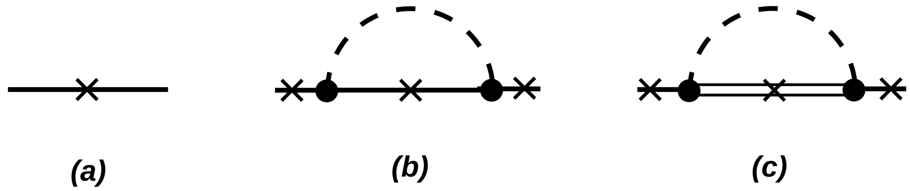

FIG. 1: Feynman diagrams contributing to the nucleon mass up to $\mathcal{O}(p^3)$ in BχPT. The internal solid lines correspond, in general, to any octet-baryon, double lines to decuplet-baryons and dashed lines to mesons. The black dots indicate 1^{st}-order couplings while crosses are insertions of $\mathcal{O}(p^2)$ operators given by the LECs b_0, b_D and b_F responsible for the leading $SU(3)_F$ breaking of the baryon-octet masses.

modern experimental determinations of $\sigma_{\pi N} \sim 60$ MeV consistent with the Zweig rule. A first indication that the decuplet contributions could help to solve the strangeness puzzle was given by the HB calculations in Ref. [26] and, indirectly, in Ref. [27] where very large and negative values of σ_s were obtained when demanding $\sigma_{\pi N} = 45$ MeV, indicating a larger σ_0.

II. CALCULATION

The expressions for the sigma terms can be obtained either from the explicit calculation of the scalar form factor of the nucleon at $q^2 = 0$ or applying the Hellmann-Feynman theorem to the chiral expansion of its mass,

$$\begin{aligned}
\sigma_{\pi N} &= \hat{m}\frac{\partial M_N}{\partial \hat{m}} = \frac{m_\pi^2}{2}\left(\frac{1}{m_\pi}\frac{\partial}{\partial m_\pi} + \frac{1}{2m_K}\frac{\partial}{\partial m_K} + \frac{1}{3m_\eta}\frac{\partial}{\partial m_\eta}\right) M_N + \mathcal{O}(p^4),\\
\sigma_s &= m_s\frac{\partial M_N}{\partial m_s} = (m_K^2 - \frac{m_\pi^2}{2})\left(\frac{1}{2m_K}\frac{\partial}{\partial m_K} + \frac{2}{3m_\eta}\frac{\partial}{\partial m_\eta}\right) M_N + \mathcal{O}(p^4).
\end{aligned} \tag{7}$$

We will follow the latter strategy since the explicit expressions for the baryon masses in the different schemes treated in this paper can be directly obtained using the Appendix of Ref. [8]. Thus, the chiral expansion of the sigma terms up to NLO from Eq. (7) is written as,

$$\begin{aligned}
\sigma_{\pi N} = &-4(2b_0 + b_D + b_F)\frac{m_\pi^2}{2} + \\
&\frac{1}{(4\pi F_\phi)^2}\sum_{\phi=\pi,K,\eta}\left(\xi^{(B)}_{N,\phi}\Sigma^{(B)}_\pi(m_\phi) + \xi^{(T)}_{N,\phi}\Sigma^{(T)}_\pi(m_\phi)\right) + \mathcal{O}(p^4),\\
\sigma_s = &-4(2b_0 + b_D - b_F)\left(m_K^2 - \frac{m_\pi^2}{2}\right) + \\
&\frac{1}{(4\pi F_\phi)^2}\sum_{\phi=\pi,K,\eta}\left(\xi^{(B)}_{N,\phi}\Sigma^{(B)}_s(m_\phi) + \xi^{(T)}_{N,\phi}\Sigma^{(T)}_s(m_\phi)\right) + \mathcal{O}(p^4).
\end{aligned} \tag{8}$$

The first line in these formulas corresponds to the LO contribution given at tree-level by the same $\mathcal{O}(p^2)$ LECs that appear in the chiral expansion of the baryon masses. While b_0 provides a $SU(3)_F$-singlet contribution, the LECs b_D and b_F induce an octet breaking (tree-level in diagram ***(a)*** in Fig. 1) which leads to a description of the baryon-octet mass splittings that is equivalent to the GMO relation [8]. The second lines enclose the NLO corrections coming, at $\mathcal{O}(p^3)$, from the loop topologies shown in Fig. 1 ***(b)*** and ***(c)***. Thus, the effect of virtual octet (B) and decuplet (T) baryons is explicitly accounted for. Their contributions are weighted by the coefficients $\xi^{(X)}_{N,\phi}$, which are combinations of $SU(3)$ Clebsch-Gordans and the meson-baryon couplings D, F (octet contributions) and $\mathcal{C}$ (decuplet contributions). The loop functions $\Sigma^{(X)}_a$ depend, exclusively, on the mass of the virtual pseudoscalar meson and on the ones of the octet and decuplet in the chiral limit, M_B and M_T respectively. Strictly speaking, the $SU(3)_F$-breaking of the baryon masses in these loops, which are represented by the crosses in Fig. 1 ***(b)*** and ***(c)***, are contributions that start at NNLO or $\mathcal{O}(p^4)$.

For the baryon masses we use the results obtained in Ref. [8] in Lorentz covariant BχPT up to $\mathcal{O}(p^3)$. The chiral loops contain divergences and analytic pieces breaking the power-counting formula [6] that are removed in dimensional regularization by the proper redefinition of the bare LECs (EOMS scheme [22]). The contributions of

the decuplet baryons are included taking the octet and decuplet masses in the chiral limit of approximately of the same order. Namely, the octet and decuplet contributions are considered on the same footing for power-counting purposes and no specific expansion in $\delta = (M_T - M_B)$ is performed. The HB formulas [26] can be always recovered from the renormalized covariant results by taking the non-relativistic expansion $M_B \sim M_T \sim \Lambda_{\chi SB}$. In particular for the decuplet, the HB results within the small-scale-expansion (SSE) [28] (which furthermore considers $\delta \sim p$) are recovered [27].

TABLE I: Values of the $\mathcal{O}(p^2)$ LECs b_D and b_F determined from the baryon octet mass splittings in the different BχPT approaches considered in this paper.

	Tree level $\mathcal{O}(p^2)$	Octet $\mathcal{O}(p^3)$		Octet+Decuplet $\mathcal{O}(p^3)$	
		HB	Covariant	HB-SSE	Covariant
b_D [GeV^{-1}]	0.060(4)	0.061(4)	0.061(4)	0.315(4)	0.161(4)
b_F [GeV^{-1}]	−0.213(2)	−0.502(2)	−0.420(2)	−0.704(2)	−0.502(2)

For the numerical values of the couplings, we use $D = 0.80$ and $F = 0.46$ [29]. The decuplet coupling $\mathcal{C}$ can be fixed from the $\Delta(1232) \to \pi N$ decay rate, giving $\mathcal{C} = 1.0$ [23]. However, there is some evidence from lattice QCD that this coupling is somewhat smaller [24]. Indeed, an $SU(3)_F$-average among the different decuplet-to-octet pionic decay channels gives $\mathcal{C} = 0.85 \pm 0.15$, that is the value we use.[3] As mentioned above, the $\mathcal{O}(p^2)$ LECs b_D and b_F are determined using the experimental baryon-octet mass splittings. Their values for the different BχPT schemes analyzed in this paper can be found in Table I [8]. For the meson decay constant we also take the $SU(3)_F$-average $F_\phi \equiv 1.17 f_\pi$ with $f_\pi = 92.4$ MeV. For the masses of the pseudoscalar mesons we use $m_\pi \equiv m_{\pi^\pm} = 139$ MeV, $m_K \equiv m_{K^\pm} = 494$ MeV, while for the baryon masses in the loops we use the chiral-limit baryon masses obtained at LO, $M_B^{(1)} = 1.151$ GeV and $M_T^{(1)} = 1.382$ GeV. The mass of the η meson is fixed with the Gell-Mann-Okubo mass relation, $3m_\eta^2 = 4m_K^2 - m_\pi^2$ which is accurate enough up to the order we work.

Finally, we will restrain our analysis to $\mathcal{O}(p^3)$ despite of the fact the extension of formulas to $\mathcal{O}(p^4)$ accuracy is straightforward and have been reported in the literature [11, 30–32]. At the latter order, 15 new LECs contribute to the baryon masses and sigma-terms. Eight of them correspond to $\mathcal{O}(p^2)$ operators which appear through diagrams with the topology of a tadpole (see Ref. [30] for details). These also contribute to the chiral expansion of the meson-baryon scattering amplitudes, although their LECs have not been determined yet from the associated experimental data or LQCD results. The other loop diagrams appearing at this order are the ones at $\mathcal{O}(p^3)$ but with the $SU(3)_F$ breaking of the baryon masses in the loop taken into account by insertions of the $\mathcal{O}(p^2)$ LECs b_0, b_D and b_F (crosses in the diagrams ***(b)*** and ***(c)*** of Fig. 1). The remaining 7 LECs correspond to $\mathcal{O}(p^4)$ operators and they renormalize the loop divergences appearing at this order. Therefore, a quantitative analysis of the sigma terms at NNLO without any further assumption on the values of the LECs (such as Large N_c constraints [31] or resonance saturation hypothesis estimates [11]) is affected, at present, by a large uncertainty. On the other hand, a promising source of theoretical information on the values of the LECs is becoming available through lattice QCD calculations. An application in this direction and performed in the EOMS scheme at $\mathcal{O}(p^4)$ can be found in [32].

Nevertheless, the analysis of part of the $\mathcal{O}(p^4)$ corrections can be useful to asses the convergence of the chiral series and to give a credible estimate on the systematic error to the $\mathcal{O}(p^3)$ results on the sigma terms due to the truncation of their chiral expansions. Indeed, we have calculated explicitly the respective corrections arising from the $SU(3)_F$ breaking of the baryon masses in the loops ***(b)*** and ***(c)*** of Fig. 1. The divergences have been renormalized in the EOMS scheme and the uncertainty on the unknown values of the $\mathcal{O}(p^4)$ LECs has been explored by varying the renormalization scale in the interval $0.7 \leq \mu \leq 1.3$ GeV. The maximal contribution obtained for these corrections in both, the octet and decuplet diagrams, is quoted as our theoretical uncertainty. That is,

$$\begin{aligned} \Delta\sigma_{\pi N}^{\rm HB} &\simeq 20 \text{ MeV}, & \Delta\sigma_s^{\rm HB} &\simeq 140 \text{ MeV}, \\ \Delta\sigma_{\pi N}^{\rm EOMS} &\simeq 6 \text{ MeV}, & \Delta\sigma_s^{\rm EOMS} &\simeq 60 \text{ MeV}. \end{aligned} \tag{9}$$

This explicit calculation of higher-order pieces already confirms the expectation that the convergence in the covariant approach is substantially better than the one obtained in the HB approach [8, 21].

[3] Note that the value for $\mathcal{C}$ of the present work is different from the one often used in HB calculations [9]. In these papers, a convention for the "vielbein" that is related to ours by a factor of 2 is employed. Moreover, the value we use in this paper is different to the one used in [8], explaining the slightly different decuplet results obtained here and there.

III. RESULTS

From the discussion above and the Eqs. (8), it is clear that the only unknown parameter in the chiral expansion of the sigma terms up to NLO is the LEC b_0. It accompanies a singlet structure that cannot be disentangled from the baryon mass term by looking at the experimental baryon masses alone [8]. The strategy followed by us in relation with Eq. (2) is that one sigma-term can be predicted if the other is taken as input. Namely, σ_0 is the pion-nucleon sigma term once b_0 is fixed by the requirement $\sigma_s = 0$. And inversely, σ_s and the strangeness content of the nucleon can be predicted using the determination of $\sigma_{\pi N}$ obtained from the πN scattering data. In Table II we show the results for σ_0 and b_0 given by the exact fulfillment of the Zweig rule and for the different BχPT schemes treated in this work. The errors in b_0 are obtained assuming an uncertainty in the Zweig rule of the order of the systematic error estimate of Eqs. (9) for σ_s. This error propagates into σ_0 and is added in quadrature with the systematic one in $\sigma_{\pi N}$.

TABLE II: Values of the $\mathcal{O}(p^2)$ LEC b_0 given by the fulfillment of the Zweig rule and the corresponding values of σ_0 for the different BχPT approaches considered in this paper.

	Tree level $\mathcal{O}(p^2)$	Octet $\mathcal{O}(p^3)$		Octet+Decuplet $\mathcal{O}(p^3)$	
		HB	Covariant	HB-SSE	Covariant
b_0^{Zweig} [GeV^{-1}]	-0.274	$-0.90(15)$	$-0.70(5)$	$-1.52(15)$	$-0.95(5)$
σ_0 [MeV]	27	58(23)	46(8)	89(23)	58(8)

As we can see in the Table II, the corrections to the LO result on σ_0 studied are large. This occurs despite that the discrepancy of the GMO equation is correctly predicted in any of these schemes and, in fact, the description of the experimental octet mass splittings improves at $\mathcal{O}(p^3)$ [8]. In other words, the empirical success of the GMO equation does not preclude Eq. (6) of receiving sizable $SU(3)_F$ breaking corrections. Another important outcome concerns the comparison between the results obtained in HB and in covariant BχPT. As already anticipated by the calculation of the $\mathcal{O}(p^4)$ pieces in Eqs. (9), the $SU(3)_F$ HB expansion has severe problems of convergence in the description of the sigma terms at $\mathcal{O}(p^3)$. The huge central value and errors of σ_0 for the HB-SSE expansion has to be regarded as a clear manifestation of these problems. Another indication in this sense is the large change in the values for σ_0 between HB and HB-SSE in Table II, a change 3 times larger than for the covariant calculation. Such large variation is at odds with the idea of resonance saturation of the LECs at $\mathcal{O}(p^2)$, indicating that higher order contributions from resonance exchanges beyond $\mathcal{O}(p^3)$ are still sizable. On the contrary, for the covariant calculation offered the difference between the calculations including/excluding the explicit decuplet of baryon resonances is only of around 10 MeV, much smaller than the difference between the LO and NLO results. This indicates a stabilization of the final outcome at the $\mathcal{O}(p^3)$ value for the covariant case. These conclusions are consistent with those derived from the analyses of other observables [8, 21, 23] that also indicate similar problems of convergence for the HB studies in the $SU(3)_F$ sector. Hence, in the rest of the paper we focus on the results obtained in covariant BχPT.

The result with only the octet contributions ought to be compared with the classical result of Gasser [3]. Notice that at the time this calculation was published χPT was in an early stage and not fully developed. Interestingly enough, this author noticed back then that using the leading non-analytic contribution (or LNAC, the old-fashioned label for the leading HB loop terms) to the baryon masses led to a very poor convergence of the respective chiral series. In order to overcome this problem, he proposed a relativistic and chiral model of the meson-baryon interaction and computed the non-analytic chiral corrections by the explicit calculation of the corresponding loop diagram, Fig. 1 ***(b)***. His approach was close, in spirit, to the one used in this paper although there are some technical differences. The most important one is that a cut-off was used for the regularization of the loop divergences. His final result, $\sigma_0 = 35(5)$ MeV, is obtained for a given value of the cut-off and it is not clear how much of the quoted uncertainty originates from reasonable variations of this parameter of the model. With these considerations, we conclude that the result obtained in our paper for the octet contribution is in reasonable agreement with the one of Gasser.

TABLE III: Value of the LEC b_0 and of the observables related to the strangeness of the nucleon, y and σ_s, obtained in covariant BχPT including decuplet contributions and using the experimental determinations of $\sigma_{\pi N}$ as input.

	$b_0^{\text{Expt.}}$ [GeV^{-1}]	y	σ_s [MeV]
$\sigma_{\pi N} = 45(7)$ MeV	$-0.79(9)$	$-0.28(13)(10)$	$-150(80)(60)$
$\sigma_{\pi N} = 59(7)$ MeV	$-0.97(9)$	$0.02(13)(10)$	$16(80)(60)$

The new contributions given by the decuplet baryons are not negligible producing a $\sim$10 MeV rise on σ_0. This result

is very important for sigma-term physics and it has a strong impact on statements about the strangeness content of the nucleon based on the value of $\sigma_{\pi N}$. In order to illustrate this point let us inverse the logic used before and determine the values for y and σ_s obtained from the two distinct experimental determinations $\sigma_{\pi N} \simeq 45$ MeV [10] and $\sigma_{\pi N} \simeq 59(7)$ MeV [16]. As we can see in Table III, the determination of the strangeness in the nucleon depends strongly on the value of the pion-nucleon sigma term due to a relative factor $\sim m_s/(2\hat{m}) \simeq 13$ between the two observables. Hence, a variation of 10 MeV in $\sigma_{\pi N}$ causes changes in σ_s spanning more than 100 MeV. This factor also amplifies the uncertainty on the latter observable that propagates from the relatively small error of the former. One has to stress that this is a general feature relative to the different quark mass entering in the definitions of the sigma terms, Eqs. (1), which makes it very difficult to give predictions of σ_s with the ballpark accuracy of few tens of MeV. Two errors are quoted for y and σ_s in the last two columns of Table III. The first stems from the propagation of the uncertainty in $\sigma_{\pi N}$ and the last from the estimated $\mathcal{O}(p^4)$ uncertainty in our calculation, Eq. (9).

Nevertheless, the most important result in Table III is that a pion-nucleon sigma term of ~ 60 MeV is perfectly consistent with an approximate fulfillment of the Zweig rule and, hence, to a small strangeness content in the nucleon. On the other hand, the canonical value of $\sigma_{\pi N}$ gives a prediction for the strangeness that is quite large and negative. It is then clear that the present calculation of σ_0 based on covariant BχPT in EOMS renormalization scheme with the decuplet as explicit degrees of freedom clearly favors relatively large values of $\sigma_{\pi N}$ with small strangeness content in the nucleon.

IV. CONCLUSIONS

In summary, we have revisited an old empirical relation between the strangeness content of the nucleon and the pion-nucleon sigma term in the context of covariant BχPT. Earlier estimates of σ_0 made at different levels of accuracy in χPT agreed on that a small violation of the Zweig rule in the nucleon requires a value of $\sigma_{\pi N}$ close to $\sim$35 MeV. A long-standing puzzle [33] has arisen from sustained experimental evidence pointing to a value of this quantity close to 60 MeV, reinforced by the values obtained from modern πN databases. We have shown that the previous calculations of σ_0 are afflicted by important systematic effects, in particular those given by the omission of the decuplet resonances. Once these are incorporated, the relatively large value of $\sigma_{\pi N}$ is consistent with and favored by the fulfillment of the Zweig rule.

V. ACKNOWLEDGMENTS

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